1

1   1 $\dfrac{1}{4}x^4 + \dfrac{2}{3}x^2 - \dfrac{10}{3}x^{\frac{3}{2}} + C$

2   $3e^x - \cos x + C$

3   $\dfrac{1}{a+1}x^{a+1} + \dfrac{1}{\ln a}a^x + C$

4   $x - \cot x + C$

5   $-2\cot x - \sec x + C$

6   $\dfrac{1}{7}x^7 - \dfrac{6}{5}x^5 + 4x^3 - 8x + C$

7   $\dfrac{1}{3}x^3 - \dfrac{1}{x} + 2x + C$

8   $\dfrac{2}{3}x^{\frac{3}{2}} + 2x + 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} - 6x^{-\frac{1}{6}} + C$

9   $\dfrac{4^x}{\ln 4} - \dfrac{1}{9^x \ln 9} + \dfrac{2}{\ln\frac{2}{3}}\left(\dfrac{2}{3}\right)^x + C$

10   $2x - \dfrac{5}{\ln\frac{2}{3}}\left(\dfrac{2}{3}\right)^x + C$

11   $\sin x - \cos x + C$

12   $2\arctan x - 3\arcsin x + C$

13   $\dfrac{4}{7}x^{\frac{7}{4}} - \dfrac{4}{15}x^{\frac{15}{4}} + C$

14   $-2\csc 2x + C$.

2        $y = \ln|x| - 2$.

3   1 $y = \dfrac{3}{4}x^{\frac{4}{3}} - x + C$

2       $y = \dfrac{3}{4}x^{\frac{4}{3}} - x + \dfrac{5}{4}$.

2

1 1 $\dfrac{1}{4}\ln|4x-3|+C$

2 $\dfrac{\sqrt{2}}{2}\arcsin\sqrt{2}x+C$

3 $\dfrac{1}{2}\ln\left|\dfrac{e^x-1}{e^x+1}\right|+C$

4 $\dfrac{1}{3}e^{3x+2}+C$

5 $\dfrac{4^x}{\ln 4}+\dfrac{9^x}{\ln 9}+\dfrac{2\cdot 6^x}{\ln 6}+C$

6 $\dfrac{\sqrt{10}}{10}\arctan\dfrac{\sqrt{10}}{2}x+C$

7 $-\dfrac{1}{5}\cos^5 x+\dfrac{2}{3}\cos^3 x-\cos x+C$

8 $\dfrac{1}{11}\tan^{11}x+C$

9 $-\dfrac{1}{16}\cos 8x-\dfrac{1}{4}\cos 2x+C$

10 $\dfrac{1}{2}x+\dfrac{1}{20}\sin 10x+C$

11 $-\dfrac{1}{x^2+4x+5}+C$

12 $-2\cos\sqrt{x}+C$

13 $-\dfrac{2}{9}(1-2x^3)^{\frac{3}{4}}+C$

14 $-\cot\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)+C$

15 $\dfrac{3}{2}(\sin x-\cos x)^{\frac{2}{3}}+C$

16 $-\dfrac{1}{\arcsin x}+C$

17 $\arctan(x-1)+C$

18 $\dfrac{1}{2}\arcsin\dfrac{2x}{3}+\dfrac{1}{4}\sqrt{9-4x^2}+C$

19    $-\ln\left|\cos\sqrt{1+x^2}\right|+C$

20    $\dfrac{1}{2}\arctan(\sin^2 x)+C$.

2   1 $\ln(\sqrt{1+e^{2x}}-1)-x+C$

2    $\ln\dfrac{\sqrt{1+x^2}-1}{|x|}+C$

3    $(\arctan\sqrt{x})^2+C$

4    $-\dfrac{1}{x\ln x}+C$

5    $\dfrac{(x+2)^{22}}{22}-\dfrac{(x+2)^{21}}{7}+C$

6    $\dfrac{(x+1)^{n+3}}{n+3}-\dfrac{2(x+1)^{n+2}}{n+2}+\dfrac{(x+1)^{n+1}}{n+1}+C$

7    $\dfrac{1}{x}(1+x^2)^{\frac{1}{2}}-\dfrac{1}{3x^3}(1+x^2)^{\frac{3}{2}}+C$

8    $\sqrt{x^2-9}-3\arccos\dfrac{3}{x}+C$

9    $\dfrac{x}{\sqrt{1-x^2}}+C$

10    $\dfrac{x}{a^2\sqrt{x^2+a^2}}+C$

11    $\sqrt{x^2-a^2}-a\ln\left|x+\sqrt{x^2-a^2}\right|+C$

12    $-\dfrac{3a+x}{2}\sqrt{x(2a-x)}+3a^2\arcsin\sqrt{\dfrac{x}{2a}}+C$

13    $\sqrt{2x}-\ln(1+\sqrt{2x})+C$

14    $-\dfrac{3}{10}(1-x)^{\frac{10}{3}}+\dfrac{6}{7}(1-x)^{\frac{7}{3}}-\dfrac{3}{4}(1-x)^{\frac{4}{3}}+C$

15    $\arccos\dfrac{1}{x}+C$

16    $-\dfrac{1}{2}x\sqrt{a^2-x^2}+\dfrac{a^2}{2}\arcsin\dfrac{x}{a}+C$

17    $-\dfrac{1}{3a^2 x^3}(a^2 - x^2)^{\frac{3}{2}} + C$

18    $\arcsin x - \tan(\dfrac{1}{2}\arcsin x) + C$

19    $-\dfrac{1}{8(x^4-1)^2} - \dfrac{3}{4(x^4-1)} + \dfrac{3}{4}\ln\left|x^4-1\right| + \dfrac{x^4}{4} + C$

20    $\dfrac{1}{n}\ln\left|\dfrac{x^n}{x^n+1}\right| + C$.

3   1 $\dfrac{1}{2}xe^{2x} - \dfrac{1}{4}e^{2x} + C$

2    $\dfrac{x^2}{2}\ln|x-1| - \dfrac{1}{2}\ln|x-1| - \dfrac{1}{4}(x+1)^2 + C$

3    $-\dfrac{(9x^2-2)\cos 3x}{27} + \dfrac{2x\sin 3x}{9} + C$

4    $-x\cot x + \ln|\sin x| + C$

5    $\dfrac{x^2}{4} + \dfrac{x\sin 2x}{4} + \dfrac{\cos 2x}{8} + C$

6    $x\arcsin x + \sqrt{1-x^2} + C$

7    $x\arctan x - \dfrac{1}{2}\ln(1+x^2) + C$

8    $\dfrac{1}{3}x^3\arctan x - \dfrac{1}{6}x^2 + \dfrac{1}{6}\ln(1+x^2) + C$

9    $x\tan x + \ln|\cos x| - \dfrac{1}{2}x^2 + C$

10    $-2\sqrt{1-x}\arcsin x + 4\sqrt{1+x} + C$

11    $x(\ln x - 1)^2 + x + C$

12    $\dfrac{1}{3}x^3\ln x - \dfrac{1}{9}x^3 + C$

13    $-\dfrac{e^{-x}(5\cos 5x + \sin 5x)}{26} + C$

14    $\dfrac{1}{10}e^x(5 - 2\sin 2x - \cos 2x) + C$

15 $\quad -\dfrac{\ln^3 x + 3\ln^2 x + 6\ln x + 6}{x} + C$

16 $\quad \dfrac{x}{2}(\sin \ln x + \cos \ln x) + C$

17 $\quad x(\arcsin x)^2 + 2\sqrt{1-x^2}\,\arcsin x - 2x + C$

18 $\quad 2e^{\sqrt{x}}(x - 2\sqrt{x} + 2) + C$

19 $\quad 2e^{\sqrt{x+1}}(\sqrt{x+1} - 1) + C$

20 $\quad x\ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C$.

4 $\quad \dfrac{(\cos x - \sin^2 x)^2}{2(1 + x\sin x)^4} + C$ $\qquad\qquad \int f(x)f'(x)dx$ $\qquad\qquad$.

5 $\quad -\ln|1 - x| - x^2 + C$.

6 $\quad -(e^{-x} + 1)\ln(1 + e^x) + x + C$.

7 $\qquad A = \displaystyle\int \frac{\cos x}{\sin x + \cos x}dx \quad B = \int \frac{\sin x}{\sin x + \cos x}dx \qquad A + B, A - B$.

8   1 $I_0 = x + C, I_1 = -\cos x + C,\ I_n = \dfrac{1}{n}[(n-1)I_{n-2} - \sin^{n-1} x\cos x]$

     2    $I_0 = x + C, I_1 = -\ln|\cos x| + C,\ I_n = \dfrac{1}{n-1}\tan^{n-1} x - I_{n-2}$

     3    $I_0 = x + C, I_1 = \ln|\sec x + \tan x| + C,\ I_n = \dfrac{1}{n-1}\left[(n-2)I_{n-2} + \dfrac{\sin x}{\cos^{n-1} x}\right]$

     4    $I_0 = -\cos x + C, I_1 = \sin x - x\cos x + C,$

        $I_n = nx^{n-1}\sin x - x^n \cos x - n(n-1)I_{n-2}$

     5    $I_0 = e^x + C, I_1 = \dfrac{1}{2}e^x(\sin x - \cos x) + C,$

        $I_n = \dfrac{1}{n^2 + 1}[n(n-1)I_{n-2} + e^x \sin^{n-1} x(\sin x - n\cos x)]$

     6    $I_0 = \dfrac{1}{\alpha + 1}x^{\alpha+1} + C,$

        $I_n = \dfrac{1}{\alpha + 1}(x^{\alpha+1}\ln^n x - nI_{n-1})$

     7    $I_0 = \arcsin x + C, I_1 = -\sqrt{1-x^2} + C,$

$$I_n = \frac{1}{n}[(n-1)I_{n-2} - x^{n-1}\sqrt{1-x^2}]$$

8   $I_0 = 2\sqrt{1+x}$ $I_1 = \ln\left|\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right| + C$ $I_n = -\frac{2n-3}{2n-2}I_{n-1} - \frac{\sqrt{1+x}}{(n-1)x^{n-1}}$

10   1 $\frac{5}{3}(x^2+x+2)^{\frac{3}{2}} + \frac{1}{4}(x+\frac{1}{2})\sqrt{x^2+x+2} + \frac{7}{16}\ln|x+\frac{1}{2}+\sqrt{x^2+x+2}| + C$

2   $\frac{1}{3}(x^2+2x-5)^{\frac{3}{2}} - (x+1)\sqrt{x^2+2x-5} + 6\ln|x+1+\sqrt{x^2+2x-5}| + C$

3   $\sqrt{x^2+x+1} - \frac{3}{2}\ln|x+\frac{1}{2}+\sqrt{x^2+x+1}| + C$

4   $-\sqrt{5+x-x^2} + \frac{5}{2}\arcsin\frac{2x-1}{\sqrt{21}} + C$.

11

$$\int \frac{a_i}{x^i} e^x dx = -\frac{a_i}{i-1} \cdot \frac{e^x}{x^{i-1}} - \frac{a_i}{(i-1)(i-2)} \cdot \frac{e^x}{x^{i-2}} - \cdots - \frac{a_i}{(i-1)!} \cdot \frac{e^x}{x} + \frac{a_i}{(i-1)!}\int \frac{e^x}{x} dx.$$

3

1   1 $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2(x+1)} + C$

2   $-\frac{1}{4}\ln|x+1| + \frac{5}{4}\ln|x-1| - \frac{1}{2}\ln(x^2+1) - \frac{3}{2}\arctan x + C$

3   $-\frac{1}{8}\ln|x+1| - 5\ln|x+2| + \frac{41}{8}\ln|x+3| - \frac{2}{x+2} - \frac{13}{4(x+3)} - \frac{3}{4(x+3)^2} + C$

4   $-\frac{1}{x+2} - \frac{3}{2}\arctan(x+2) - \frac{x+2}{2(x^2+4x+5)} + C$

5   $\frac{1}{2}\ln\frac{(x+1)^2}{x^2-x+1} + \sqrt{3}\arctan\frac{2x-1}{\sqrt{3}} + C$

6   $\frac{1}{4}\ln\frac{x^2+x+1}{x^2-x+1} + \frac{1}{2\sqrt{3}}\arctan\frac{\sqrt{3}x}{1-x^2} + C$

7   $\frac{x^3}{3} - \frac{5}{2}x^2 + 21x - 80\ln|x+4| + C$

8   $x + \frac{1}{8}\ln\frac{(x-1)^2}{x^2+x+6} - \frac{43}{4\sqrt{23}}\arctan\frac{2x+1}{\sqrt{23}} + C$

9    $\dfrac{1}{4}\ln\left|\dfrac{x+1}{x-1}\right| - \dfrac{1}{2}\arctan x + C$

10    $\dfrac{\sqrt{2}}{8}\ln\dfrac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1} + \dfrac{\sqrt{2}}{4}\arctan(\sqrt{2}x+1) + \dfrac{\sqrt{2}}{4}\arctan(\sqrt{2}x-1) + C$

11    $\dfrac{1}{2}\ln\dfrac{x^2+x+1}{x^2+1} + \dfrac{1}{\sqrt{3}}\arctan\dfrac{2x+1}{\sqrt{3}} + C$

12    $-\ln|x| + \dfrac{2}{3}\ln|x-1| + \dfrac{1}{6}\ln(x^2+x+1) + \dfrac{1}{\sqrt{3}}\arctan\dfrac{2x+1}{\sqrt{3}} + C$

13    $\dfrac{4}{\sqrt{3}}\arctan\dfrac{2x+1}{\sqrt{3}} + \dfrac{x+1}{x^2+x+1} + C$

14    $\ln|x| - \dfrac{2}{7}\ln|1+x^7| + C$

15    $-\dfrac{x^5+2}{10(x^{10}+2x^5+2)} - \dfrac{1}{10}\arctan(x^5+1) + C$

16    $-\dfrac{x^n}{2n(x^{2n}+1)} + \dfrac{1}{2n}\arctan x^n + C$.

4   1 $\dfrac{1}{6}(x-1)\sqrt{2+4x} + C$

     2    $2\arcsin\sqrt{\dfrac{x-a}{b-a}} + C$

     3    $-\dfrac{1}{4}(2x+3)\sqrt{1+x-x^2} + \dfrac{7}{8}\arcsin\dfrac{2x-1}{\sqrt{5}} + C$

     4    $\ln\left|\dfrac{x^2-1+\sqrt{x^4+1}}{x}\right| + C$

     5    $\dfrac{1}{2}x^2 - \dfrac{1}{2}x\sqrt{x^2-1} + \dfrac{1}{2}\ln(x+\sqrt{x^2+1}) + C$

     6    $\ln\left(1+\sqrt{\dfrac{x+1}{x-1}}\right) - \ln\left|\sqrt{\dfrac{x+1}{x-1}}-1\right| + \sqrt{x^2-1} + C$

     7    $2\ln(\sqrt{1+x}+\sqrt{x}) + C$

     8    $\dfrac{2x^2-1}{3x^3}\sqrt{x^2+1} + C$

9    $2\sqrt{x} - 4\sqrt[4]{x} + 4\ln(\sqrt[4]{x} + 1) + C$

10    $\dfrac{3}{25}\left(\dfrac{x-4}{x+1}\right)^{\frac{5}{3}} + C$

11    $-\dfrac{3}{2}\ln(\sqrt[3]{x+1} - \sqrt[3]{x-2}) - \sqrt{3}\arctan\dfrac{\sqrt[3]{x+1} + 2\cdot\sqrt[3]{x-2}}{\sqrt{3}\cdot\sqrt[3]{x+1}} + C$

12    $\dfrac{1}{4}\ln\left|\dfrac{\sqrt[4]{1+x^4} - 1}{\sqrt[4]{1+x^4} + 1}\right| + \dfrac{1}{2}\arctan\sqrt[4]{1+x^4} + C$.

5         $\sqrt{a+x} = t$ $\int R(x, \sqrt{a+x}, \sqrt{b+x})dx = \int R_1(t, \sqrt{t^2+c})dt$

      $\sqrt{t^2+c} = t + u$.

6   1 $\dfrac{1}{3}\ln\left|\dfrac{\tan\dfrac{x}{2} + 3}{\tan\dfrac{x}{2} - 3}\right| + C$

     2    $\dfrac{2\sqrt{3}}{3}\arctan\dfrac{2\tan\dfrac{x}{2} + 1}{\sqrt{3}} + C$

     3    $\dfrac{\sqrt{3}}{6}\arctan\left(\dfrac{1}{\sqrt{3}}\tan\dfrac{x}{2}\right) + \dfrac{\sqrt{3}}{6}\arctan\left(\sqrt{3}\tan\dfrac{x}{2}\right) + C$

     4    $\ln\left|\tan\dfrac{x}{2} + 1\right| + C$

     5    $\dfrac{1}{\sqrt{5}}\arctan\dfrac{3\tan\dfrac{x}{2} + 1}{\sqrt{5}} + C$

     6    $\dfrac{1}{6}\ln\dfrac{(1-\cos x)(2+\cos x)^2}{(1+\cos x)^3} + C$

     7    $\dfrac{1}{2}\ln\left|\tan\dfrac{x}{2}\right| - \dfrac{1}{4}\tan^2\dfrac{x}{2} + C$

     8    $\dfrac{1}{\cos(a-b)}\ln\left|\dfrac{\sin(x+a)}{\cos(x+b)}\right| + C$

     9    $\cot a \cdot \ln\left|\dfrac{\cos x}{\cos(x+a)}\right| - x + C$

10 $\quad \dfrac{1}{2}(\sin x - \cos x) - \dfrac{1}{2\sqrt{2}} \ln\left|\tan(\dfrac{x}{2} + \dfrac{\pi}{8})\right| + C$

11 $\quad -2\cot 2x + C$

12 $\quad x - \dfrac{1}{\sqrt{2}} \arctan(\sqrt{2}\,\tan x) + C$.

7 $\quad$ 1 $\quad \dfrac{1}{x+1} e^x + C$

2 $\quad \dfrac{x \ln x}{\sqrt{1+x^2}} - \ln(x + \sqrt{1+x^2}) + C$

3 $\quad x \ln^2(1 + \sqrt{1+x^2}) - 2\sqrt{1+x^2}\,\ln(x + \sqrt{1+x^2}) + 2x + C$

4 $\quad \dfrac{2}{3} x^{\frac{3}{2}} \ln^2 x - \dfrac{8}{9} x^{\frac{3}{2}} \ln x + \dfrac{16}{27} x^{\frac{3}{2}} + C$

5 $\quad \dfrac{1}{2} e^x \left[(x^2 - 1)\sin x - (x-1)^2 \cos x\right] + C$

6 $\quad x \ln(1+x^2) - 2x + 2\arctan x + C$

7 $\quad \dfrac{1}{4}(\arcsin x)^2 - \dfrac{1}{2} x\sqrt{1-x^2}\,\arcsin x + \dfrac{1}{4} x^2 + C$

8 $\quad -\dfrac{2}{\sqrt{3}} \arctan\sqrt{\dfrac{3x+3}{x-3}} + C$

9 $\quad (x+1)\arctan\sqrt{x} - \sqrt{x} + C$

10 $\quad 4\sqrt{x}\sin\sqrt{x} - 2(x-2)\cos\sqrt{x} + C$

11 $\quad x\tan\dfrac{x}{2} + C$

12 $\quad \dfrac{\sqrt{2}}{2} \ln\left|\dfrac{\sqrt{1+\sin x} + \sqrt{2}}{\sqrt{1+\sin x} - \sqrt{2}}\right| + C$

13 $\quad \dfrac{1}{2}\sec x \tan x - \dfrac{1}{2} \ln\left|\sec x + \tan x\right| + C$

14 $\quad (x - \sec x)e^{\sin x} + C$

15 $\quad \dfrac{1}{2} \ln\left|\dfrac{e^x - 1}{e^x + 1}\right| + C$

16 $\quad \dfrac{1}{ab} \arctan\dfrac{a\tan x}{b} + C$

17 $6\ln\dfrac{\sqrt[6]{x}}{\sqrt[6]{x}+1}+C$

18 $\dfrac{x^2-1}{2}\ln\dfrac{1+x}{1-x}+x+C$

19 $-\dfrac{1}{4}x^2+\dfrac{1}{4}(\arcsin x)^2+\dfrac{1}{2}x\sqrt{1-x^2}\,\arcsin x+C$

20 $x+\dfrac{1}{1+e^x}-\ln(1+e^x)+C$.