

[书名]: Advanced Calculus (Second Edition) 高等微积分 (英文版·第二版)

[作者]: (美) Patrick M. Fitzpatrick

[出版商]: 机械工业出版社

[页数]: 590

[适用范围]: 适合数学系与理工科其他专业的本科生

[预备知识]: 高中数学

[习题数量]: 较大

[习题难度]: 具有一定难度

[推荐强度]: 9.3/10

#### [书籍评价]

本书以清晰、简洁的方式介绍了数学分析的基本概念: 第一部分讲述单变量函数的微积分, 包括实数理论、数列的收敛、函数的连续性和极限、函数的导数和积分、多项式逼近等; 第二部分把微积分的概念推广到多维欧几里得空间, 讨论多变量函数的偏导数、反函数、隐函数及其应用、曲线积分和曲面积分等。

数学分析已经根植于自然科学和社会科学的各个学科分支之中, 微积分作为数学分析的基础, 不仅要为全部数学方法和算法工具提供方法论, 同时还要为人们灌输逻辑思维的方法, 本书在实现这一目标中取得了引人注目的成果。本书一方面按传统的和严格的演绎形式介绍微积分的所有主题, 另一方面强调主题的相关性和统一性, 使读者受到数学科学思维的系统训练。

本书的一大特点是除了包含必不可少的论题, 如实数、收敛序列、连续函数与极限、初等函数、微分、积分、多元函数微积分等以外, 还包含其他一些重要的论题, 如求积分的逼近方法、Weierstrass 逼近定理、度量空间等。例如本书专门用一章讨论度量空间, 从而把在欧几里得空间讨论微积分时使用的许多概念和导出的结果扩展到更抽象的空间中, 引导读者作广泛深入的思考。

另外, 与第一版相比, 第二版增加了 200 多道难易不等的习题。全书贯穿了许多具有启发性的例题, 并且本版还为教学考虑进行了许多实质性的改动, 例如将选学材料与前后内容的关联度降到最低, 单独放置, 既不影响教学和读者自学的进度, 又能让读者集中攻破一些难点, 这样使得全书的叙述更简洁、更自然。本书曾于 2003-2004 年作为马里兰大学教材。

#### [作者简介]

Patrick M. Fitzpatrick 拥有格兰特大学博士学位, 是纽约大学科朗研究所和芝加哥大学的博士后, 1975 年进入马里兰大学 College Park 分校任教, 现在是数学系教授和系主任, 同时它还是巴黎大学和佛罗伦萨大学的客座教授。他的研究方向是非线性泛函分析, 在该方向著有 50 多篇论文。

(高威)

#### [目录]

Preface

Preliminaries

1 TOOLS FOR ANALYSIS

- 1.1 The Completeness Axiom and Some of Its Consequences
- 1.2 The Distribution of the Integers and the Rational Numbers
- 1.3 Inequalities and Identities
- 2 CONVERGENT SEQUENCES
  - 2.1 The Convergence of Sequences
  - 2.2 Sequences and Sets
  - 2.3 The Monotone Convergence Theorem
  - 2.4 The Sequential Compactness Theorem
  - 2.5 Covering Properties of Sets\*
- 3 CONTINUOUS FUNCTIONS
  - 3.1 Continuity
  - 3.2 The Extreme Value Theorem
  - 3.3 The Intermediate Value Theorem
  - 3.4 Uniform Continuity
  - 3.5 The  $\varepsilon$ - $\delta$  Criterion for Continuity
  - 3.6 Images and Inverses; Monotone Functions
  - 3.7 Limits
- 4 DIFFERENTIATION
  - 4.1 The Algebra of Derivatives
  - 4.2 Differentiating Inverses and Compositions
  - 4.3 The Mean Value Theorem and Its Geometric Consequences
  - 4.4 The Cauchy Mean Value Theorem and Its Analytic Consequences
  - 4.5 The Notation of Leibnitz
- 5\* ELEMENTARY FUNCTIONS AS SOLUTIONS OF DIFFERENTIAL EQUATIONS
  - 5.1 Solutions of Differential Equations
  - 5.2 The Natural Logarithm and Exponential Functions
  - 5.3 The Trigonometric Functions
  - 5.4 The Inverse Trigonometric Functions
- 6 INTEGRATION: TWO FUNDAMENTAL THEOREMS
  - 6.1 Darboux Sums; Upper and Lower Integrals
  - 6.2 The Archimedes-Riemann Theorem
  - 6.3 Additivity, Monotonicity, and Linearity
  - 6.4 Continuity and Integrability
  - 6.5 The First Fundamental Theorem: Integrating Derivatives

- 6.6 The Second Fundamental Theorem: Differentiating integrals
- 7\* INTEGRATION: FURTHER TOPICS
  - 7.1 Solutions of Differential Equations
  - 7.2 Integration by Parts and by Substitution
  - 7.3 The Convergence of Darboux and Riemann Sums
  - 7.4 The Approximation of Integrals
- 8 APPROXIMATION BY TAYLOR POLYNOMIALS
  - 8.1 Taylor Polynomials
  - 8.2 The Lagrange Remainder Theorem
  - 8.3 The Convergence of Taylor Polynomials
  - 8.4 A Power Series for the Logarithm
  - 8.5 The Cauchy Integral Remainder Theorem
  - 8.6 A Nonanalytic, Infinitely Differentiable Function
  - 8.7 The Weierstrass Approximation Theorem
- 9 SEQUENCES AND SERIES OF FUNCTIONS
  - 9.1 Sequences and Series of Numbers
  - 9.2 Pointwise Convergence of Sequences of Functions
  - 9.3 Uniform Convergence of Sequences of Functions
  - 9.4 The Uniform Limit of Functions
  - 9.5 Power Series
  - 9.6 A Continuous Nowhere Differentiable Function
- 10 THE EUCLIDEAN SPACE  $\mathbb{R}^n$ 
  - 10.1 The Linear Structure of  $\mathbb{R}^n$  and the Scalar Product
  - 10.2 Convergence of Sequences in  $\mathbb{R}^n$
  - 10.3 Open Sets and Closed Sets in  $\mathbb{R}^n$
- 11 CONTINUITY, COMPACTNESS, AND CONNECTEDNESS
  - 11.1 Continuous Functions and Mappings
  - 11.2 Sequential Compactness, Extreme Values, and Uniform Continuity
  - 11.3 Pathwise Connectedness and the Intermediate Value Theorem\*
  - 11.4 Connectedness and the Intermediate Value Property\*
- 12\* METRIC SPACES
  - 12.1 Open Sets, Closed Sets, and Sequential Convergence
  - 12.2 Completeness and the Contraction Mapping Principle
  - 12.3 The Existence Theorem for Nonlinear Differential Equations

- 12.4 Continuous Mappings between Metric Spaces
- 12.5 Sequential Compactness and Connectedness
- 13 DIFFERENTIATING FUNCTIONS OF SEVERAL VARIABLES
  - 13.1 Limits
  - 13.2 Partial Derivatives
  - 13.3 The Mean Value Theorem and Directional Derivatives
- 14 LOCAL APPROXIMATION OF REAL-VALUED FUNCTIONS
  - 14.1 First-Order Approximation, Tangent Planes, and Affine Functions
  - 14.2 Quadratic Functions, Hessian Matrices, and Second Derivatives\*
  - 14.3 Second-Order Approximation and the Second-Derivative Test\*
- 15 APPROXIMATING NONLINEAR MAPPINGS BY LINEAR MAPPINGS
  - 15.1 Linear Mappings and Matrices
  - 15.2 The Derivative Matrix and the Differential
  - 15.3 The Chain Rule
- 16 IMAGES AND INVERSES: THE INVERSE FUNCTION THEOREM
  - 16.1 Functions of a Single Variable and Maps in the Plane
  - 16.2 Stability of Nonlinear Mappings
  - 16.3 A Minimization Principle and the General Inverse Function Theorem
- 17 THE IMPLICIT FUNCTION THEOREM AND ITS APPLICATIONS
  - 17.1 A Scalar Equation in Two Unknowns: Dini's Theorem
  - 17.2 The General Implicit Function Theorem
  - 17.3 Equations of Surfaces and Paths in  $\mathbb{R}^3$
  - 17.4 Constrained Extrema Problems and Lagrange Multipliers
- 18 INTEGRATING FUNCTIONS OF SEVERAL VARIABLES
  - 18.1 Integration of Functions on Generalized Rectangles
  - 18.2 Continuity and Integrability
  - 18.3 Integration of Functions on Jordan Domains
- 19 ITERATED INTEGRATION AND CHANGES OF VARIABLES
  - 19.1 Fubini's Theorem
  - 19.2 The Change of Variables Theorem: Statements and Examples
  - 19.3 Proof of the Change of Variables Theorem
- 20 LINE AND SURFACE INTEGRALS
  - 20.1 Arclength and Line Integrals
  - 20.2 Surface Area and Surface Integrals

20.3 The Integral Formulas of Green and Stokes

A CONSEQUENCES OF THE FIELD AND POSITIVITY AXIOMS

A.1 The Field Axioms and Their Consequences

A.2 The Positivity Axioms and Their Consequences

B LINEAR ALGEBRA

Index